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The EPR Paradox, Einstein-Rosen bridges and teleportation

Abstract

In this review, we go over the bases of quantum teleportation, ER bridges in General relativity, and the foundational work on the hypothesis ER=EPR and summarize the resulting wormhole teleportation protocol. We then discuss that — resulting from ER=EPR — certain wormholes have to be either traversable or at the very least let information permeate, resulting in the exploration of the possibility that incoming matter might change the metric outside of wormhole throats. In this study, made in the Schwarzschild metric with the original coordinate system, we managed to find a non-zero energy-momentum tensor produced by a particular solution of the electromagnetic wave equation in curved spacetime, implying a change in the overall metric by Einstein's Equation.

1 Introduction

Quantum mechanics and general relativity have both led to the discovery of phenomena that seem to connect two arbitrarily distant entities: for quantum mechanics the EPR Paradox formulated by its namesakes Einstein, Podolski, and Rosen [1] connects two distant systems, through what is commonly called a quantum entanglement, such that the measurements on one instantaneously affects the other; for General Relativity, two black holes can be connected by a wormhole, or Einstein Rosen bridge, first described by extending the Schwarzschild solution to Einstein's equations [2]. In more recent years, several papers have linked both phenomena together, culminating in the elaboration of the hypothesis ER=EPR [3], that is that quantum entanglements and wormholes represent the same object. It is thus logical that, since Quantum Teleportation is possible [4] according to ER=EPR, teleportation using wormholes is also possible [5]. However, the hypothesis ER=EPR leads to interesting implications on the possibility of traversability or at least information permeability of wormholes, which we will elaborate on. We chose in this paper to explore the possibility, contrary to the no-hair theorem, that incoming information in one end of the wormhole changes the metric on the other end.

Firstly, this paper will review the relevant concepts surrounding the EPR Paradox and ER bridges in order to understand ER=EPR. Then, we will summarize the claim ER=EPR, as well as the wormhole teleportation protocol. We will then start with the author's contributions by quickly advancing and explaining the claim that certain wormholes have to be either traversable or permeable if ER=EPR, and by trying to see how an incoming photon might change the metric of a Schwarzschild black hole. To do this, we will find a particular solution of the electromagnetic wave equation in curved spacetime and deduce the energy-momentum tensor from it. Due to a lack of time for this project, the author only managed to derive the altered energy-momentum tensor, which by being non-zero is sufficient to prove a change in the metric. They also weren't able to solve Einstein's equation for more specific results, nor to study this phenomenon in the more general Kruskal-Szekeres coordinates for more decisive results.

2 The EPR paradox, quantum teleportation and Einstein-Rosen bridges

Before diving directly into ER=EPR, we first want to review some useful concepts, namely the EPR Paradox and ER Bridges. As we also want to study the wormhole teleportation protocol, we will also have to review the (very similar) quantum teleportation protocol. But first, we might want a little refresher on how quantum mechanics work by using the example of multiple SG devices in a row. You can skip directly to subsection 2.2.

2.1 Reminder on quantum mechanics: multiple SG devices in a row

A Stern-Gerlach device oriented in the $\vec{n}$ direction, in short an SG_n device, is composed of two magnets creating a magnetic field pointing in the $\vec{n}$ direction. This device can perform spin measurements on particles with quantum behaviours. Let's consider the following exercise: A particle with spin s passes through an SG_z device and is found to have spin value $S_z = s\hbar$. It now travels through an SG_x device. What are the possible values of S_x and what is their probability of appearing? We will solve this for $s = 1/2$.

We know that the possible values of S_x are $\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$. We also know that the eigenstates of $\hat{S}_x$ are:

$$\begin{aligned}\left|\frac{1}{2},\frac{1}{2}\right\rangle_x &= \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2},\frac{1}{2}\right\rangle + \left|\frac{1}{2},-\frac{1}{2}\right\rangle\right)\\ \left|\frac{1}{2},-\frac{1}{2}\right\rangle_x &= \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2},\frac{1}{2}\right\rangle - \left|\frac{1}{2},-\frac{1}{2}\right\rangle\right).\end{aligned}$$

Since $S_z = \frac{\hbar}{2}$, the particle is initially in state $\left|\frac{1}{2},\frac{1}{2}\right\rangle$, hence:

$$\begin{aligned}P\left(S_x = \frac{\hbar}{2}\right) &= \left|{}_x\left\langle \frac{1}{2},\frac{1}{2} \mid \frac{1}{2},\frac{1}{2}\right\rangle_z\right|^2 = \frac{1}{2}\\ P\left(S_x = -\frac{\hbar}{2}\right) &= \left|{}_x\left\langle \frac{1}{2},\frac{1}{2} \mid \frac{1}{2},\frac{1}{2}\right\rangle_z\right|^2 = \frac{1}{2}.\end{aligned}$$

Now we can finally start reviewing the EPR Paradox.

2.2 The EPR Paradox and the Bell Inequality

The EPR Paradox was first formulated by Einstein, Podolski and Rosen [1] and concerned the two observables position and momentum. The following version of the paradox was proposed by Bohm [6]:

Suppose a particle in spin state $|0,0\rangle$ decays into 2 spin-$\frac{1}{2}$ particles. Due to conservation of linear and angular momentum, they move in opposite directions and their spin along any given direction must be opposite. Suppose now that two observers with SG devices, which we call A and B, will measure the spin of their respective particle along the axes $\overrightarrow{n_A}$ and $\overrightarrow{n_B}$. Since we know that:

$$|0,0\rangle = \frac{1}{\sqrt{2}}|+z,-z\rangle - \frac{1}{\sqrt{2}}|-z,+z\rangle \tag{2.1}$$
$$= \frac{1}{\sqrt{2}}|+x,-x\rangle - \frac{1}{\sqrt{2}}|-x,+x\rangle \tag{2.2}$$

then we can conclude that, if $\overrightarrow{n_A} = \overrightarrow{n_B} = \overrightarrow{k}$, if A measures $S_{1z} = \frac{\hbar}{2}$, then B measures $S_{2z} = -\frac{\hbar}{2}$, and if A measures $S_{1z} = -\frac{\hbar}{2}$, then B measures $S_{2z} = \frac{\hbar}{2}$. But likewise, if if $\overrightarrow{n_A} = \overrightarrow{n_B} = \overrightarrow{i}$, then if $S_{1x} = \frac{\hbar}{2}$, then $S_{2x} = -\frac{\hbar}{2}$, and if $S_{1x} = -\frac{\hbar}{2}$, then $S_{2x} = \frac{\hbar}{2}$.

Yet, if $\overrightarrow{n_A} = \overrightarrow{k}, \overrightarrow{n_B} = \overrightarrow{i}$, if $S_{1z} = \frac{\hbar}{2}$, then there is a 50% chance that $S_{2x} = \frac{\hbar}{2}$ and a 50% chance that $S_{2x} = -\frac{\hbar}{2}$. We can only determine the value of S_{2x} by measuring it.

This would mean that we can determine with no uncertainty both the S_z and S_x values of the second particle, which violates the uncertainty principle if local realism, i.e. the assumption that the spin of each particle has an intrinsic and defined value, holds.

To prove that local realism does not hold, we use the Bell Inequality[7]:

Theorem 1 (Bell Inequality). *Suppose local realism holds. Suppose also that a collection of particles in spin state $|0,0\rangle$ decay into pairs of spin-$\frac{1}{2}$ particles, with possibles values of spin $\left\{\frac{\hbar}{2}, -\frac{\hbar}{2}\right\}$ along 3 possible directions $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$. If the particles of each pair pass the SG devices A and B, respectively, oriented independently along $\overrightarrow{a}$, $\overrightarrow{b}$ or $\overrightarrow{c}$, and the event $(\pm n_1; \pm n_2)$ describes the situation in which the particle measured by A has spin value $\pm\frac{\hbar}{2}$ along $\overrightarrow{n_1}$ and the particle measured by B has spin value $\pm\frac{\hbar}{2}$ along $\overrightarrow{n_2}$, then:*

$$P(+a;+b) = P(+a;+c) + P(+c;+b). \tag{2.3}$$

Using this, then, if $\overrightarrow{c}$ bisects $\overrightarrow{a}$ and $\overrightarrow{b}$ by an angle $\theta/2$, then, by the predictions of quantum mechanics and (2.3), one finds

$$\sin^2\theta \leq 2\sin^2(\theta/2) \tag{2.4}$$

for all $\theta \in \mathbb{R}$, yet (2.4) only holds for $0 < \theta < \frac{\pi}{2}$. Thus local realism and quantum mechanics fundamentally yield different results. Experiments measuring the polarization states of pairs of photons yield results in accordance with QM, but violating Bell's inequality. Hence Bell's inequality disproves local realism and the existence of local hidden variables.

The EPR Paradox, under the form of quantum entanglement, can be used for teleportation, as we will now see.

2.3 Quantum Teleportation

A protocol to teleport quantum states was first elaborated in 1993 [4], derived directly from the EPR Paradox. The following discussion is based on it.

Suppose Alice has a particle in state $|\phi_1\rangle$. She wishes to send Bob enough information to make a copy of the state. Unless $|\phi_1\rangle$ is an already known eigenstate, Alice cannot determine $|\phi_1\rangle$ through measurement. Alice could solve the given task in different ways: either by trivially sending Bob the particle directly, or through a spin-exchange measurement. A spin-exchange measurement consists of making the particle in state $|\phi_1\rangle$ interact with another one, called ancilla, in state $|a_0\rangle$ through a unitary operation, thus leaving the initial particle in a new state $|\phi_0\rangle$. The ancilla, now in state $|a_1\rangle$, contains all the information necessary for Bob to reverse the steps and obtain $|\phi_1\rangle$. This example demonstrates the no-cloning principle of quantum information, stating that no quantum state $|\psi\rangle$ can be copied without being destroyed.

It is possible for Alice to divide the information needed to recreate $|\phi_1\rangle$ into a classical part and a quantum part, and upon reception of both, Bob can reconstruct $|\phi_1\rangle$, destroying Alice's $|\phi_1\rangle$ in the process. This process is called quantum teleportation, and, because of the need of a classical information channel, the "teleportation" is not instantaneous. The full process for the teleportation of $|\phi_1\rangle$ for a spin-$\frac{1}{2}$ particle is the following:

Two spin-$\frac{1}{2}$ particles, numbered 2 and 3, pertaining to an EPR singlet are in the overall state

$$\left|\psi_{23}^{(-)}\right\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_2\rangle|\uparrow_3\rangle\right) \tag{2.5}$$

as in (2.1). Alice's particle shall be numbered 1. Particle 2 is received by Alice, while particle 3 is received by Bob. As the systems (1) and (23) are uncorrelated, we get that the overall system is given by

$$|\psi_{123}\rangle = |\phi_1\rangle\left|\psi_{23}^{(-)}\right\rangle. \tag{2.6}$$

Alice now entangles particles 1 and 2 by measuring an observable of the system (12), i.e performing a von Neumann measurement, in the Bell basis:

$$\left|\psi_{12}^{(\pm)}\right\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle|\downarrow_2\rangle \pm |\downarrow_1\rangle|\uparrow_2\rangle\right) \tag{2.7}$$
$$\left|\phi_{12}^{(\pm)}\right\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle|\uparrow_2\rangle \pm |\downarrow_1\rangle|\downarrow_2\rangle\right) \tag{2.8}$$

which is orthonormal. Hence:

$$|\uparrow_1\rangle|\downarrow_2\rangle = \frac{1}{\sqrt{2}}\left(\left|\psi_{12}^{(+)}\right\rangle + \left|\psi_{12}^{(-)}\right\rangle\right) \tag{2.9}$$
$$|\downarrow_1\rangle|\uparrow_2\rangle = \frac{1}{\sqrt{2}}\left(\left|\psi_{12}^{(+)}\right\rangle - \left|\psi_{12}^{(-)}\right\rangle\right) \tag{2.10}$$
$$|\uparrow_1\rangle|\uparrow_2\rangle = \frac{1}{\sqrt{2}}\left(\left|\phi_{12}^{(+)}\right\rangle + \left|\phi_{12}^{(-)}\right\rangle\right) \tag{2.11}$$
$$|\downarrow_1\rangle|\downarrow_2\rangle = \frac{1}{\sqrt{2}}\left(\left|\phi_{12}^{(+)}\right\rangle - \left|\phi_{12}^{(-)}\right\rangle\right) \tag{2.12}$$

If we write, for convenience,

$$|\phi_1\rangle = a|\uparrow_1\rangle + b|\downarrow_1\rangle \tag{2.13}$$

with $|a|^2 + |b|^2 = 1$, we get:

$$\begin{aligned}|\psi_{123}\rangle &= |\phi_1\rangle\left|\psi_{23}^{(-)}\right\rangle \\ &= \left(a|\uparrow_1\rangle + b|\downarrow_1\rangle\right)\left(\frac{1}{\sqrt{2}}\left(|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_2\rangle|\uparrow_3\rangle\right)\right) \\ &= \frac{a}{\sqrt{2}}\left(|\uparrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\uparrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle\right) \\ &\quad + \frac{b}{\sqrt{2}}\left(|\downarrow_1\rangle|\uparrow_2\rangle|\downarrow_3\rangle - |\downarrow_1\rangle|\downarrow_2\rangle|\uparrow_3\rangle\right)\end{aligned} \tag{2.14}$$

hence, by (2.9)-(2.12):

$$\begin{aligned}|\psi_{123}\rangle = \frac{1}{2}&\left[\left|\psi_{12}^{(-)}\right\rangle\left(-a|\uparrow_3\rangle - b|\downarrow_3\rangle\right) + \left|\psi_{12}^{(+)}\right\rangle\left(-a|\uparrow_3\rangle + b|\downarrow_3\rangle\right)\right] \\ &+ \frac{1}{2}\left[\left|\phi_{12}^{(-)}\right\rangle\left(a|\downarrow_3\rangle + b|\uparrow_3\rangle\right) + \left|\phi_{12}^{(+)}\right\rangle\left(a|\downarrow_3\rangle - b|\uparrow_3\rangle\right)\right].\end{aligned} \tag{2.15}$$

Therefore, all 4 outcomes are equally likely with probability $1/4$. After Alice's measurement, Bob's particle 3 will take one of the following states:

$$\begin{aligned}|\phi_3\rangle \equiv -\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}|\phi_3\rangle \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}|\phi_3\rangle, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}|\phi_3\rangle.\end{aligned} \tag{2.16}$$

Each of these can be obtained by applying a unitary operation on $|\phi_1\rangle$: $|\phi_3\rangle$ is just $|\phi_1\rangle$ times a phase, whilst the other operations are clockwise rotations of 180° around the z, x and y axes, respectively, hence Bob just needs to rotate them anti-clockwise to obtain $|\phi_1\rangle$ again. All Bob needs to replicate $|\phi_1\rangle$ is the information, sent by Alice, of which of the four states $\left|\psi_{12}^{(\pm)}\right\rangle$

and $\left|\phi_{12}^{(\pm)}\right\rangle$ she obtained. No trace of $|\phi_1\rangle$ is left in her results, and she just needs to send the information to Bob. At the end of the process, two bits of information, uncorrelated to $|\phi_1\rangle$, are left behind.

Quantum teleportation also works for mixed or entangled states. If, taking the same example, we start with particle 1 already forming an EPR singlet with a fourth particle, which we call 0, i.e.

$$\left|\psi_{01}^{(-)}\right\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow_0\rangle |\downarrow_1\rangle - |\downarrow_0\rangle |\uparrow_1\rangle \right), \tag{2.17}$$

then

$$\begin{aligned} |\psi_{0123}\rangle &= \left|\psi_{01}^{(-)}\right\rangle \left|\psi_{23}^{(-)}\right\rangle \\ &= \frac{1}{2} \big[|\uparrow_0\rangle |\downarrow_1\rangle |\uparrow_2\rangle |\downarrow_3\rangle - |\uparrow_0\rangle |\downarrow_1\rangle |\downarrow_2\rangle |\uparrow_3\rangle \\ &\quad - |\downarrow_0\rangle |\uparrow_1\rangle |\uparrow_2\rangle |\downarrow_3\rangle + |\downarrow_0\rangle |\uparrow_1\rangle |\downarrow_2\rangle |\uparrow_3\rangle \big] \\ &= \frac{1}{2} \Big[\left|\psi_{12}^{(-)}\right\rangle \left|\psi_{03}^{(-)}\right\rangle + \left|\psi_{12}^{(+)}\right\rangle \left|\psi_{03}^{(+)}\right\rangle + \left|\phi_{12}^{(-)}\right\rangle \left|\phi_{03}^{(-)}\right\rangle \\ &\quad + \left|\phi_{12}^{(+)}\right\rangle \left|\phi_{03}^{(+)}\right\rangle \Big] \end{aligned} \tag{2.18}$$

hence 0 and 3 form a singlet after the measurement on system (12).

All these results can be generalized to systems having N>2 orthogonal states. Alice would use the pair (23) of N-state particles in a completely entangled state, which can be written as:

$$\sum_j \frac{1}{\sqrt{N}} |j\rangle \otimes |j\rangle \tag{2.19}$$

where $j = 0, 1, ..., N-1$ are the N states. Let the state of particle 1 which we want to teleport be:

$$|\phi\rangle = \sum_j \rho_j |j\rangle, \tag{2.20}$$

with $\sum_j |\rho_j|^2 = 1$. Alice then performs her measurement on the system (12), yielding one of the following:

$$|\psi_{nm}\rangle = \sum_j \frac{1}{\sqrt{N}} e^{\frac{2\pi i j n}{N}} |j\rangle \otimes |(j+m) \bmod N\rangle, \tag{2.21}$$

where $n, m \in \{0, 1, ..., N-1\}$, the state of the total system being:

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_j \rho_j \sum_k |j\rangle \otimes |k\rangle \otimes |k\rangle \tag{2.22}$$

Again, after performing a measurement on her particle and one of the two entangled particles, Alice sends classical information to Bob. This time, her information is of at least 2^N bits. Bob then performs the unitary transformation:

$$U_{nm} = \sum_k e^{\frac{2\pi i k n}{N}} |k\rangle \langle (k+m) \bmod N| \tag{2.23}$$

on his particle to obtain $|\phi\rangle$ again.

The message sent by Alice is crucial. If Bob were to guess her outcome, the sate $|\phi\rangle$ would be reconstructed (in the spin-$\frac{1}{2}$ case) as a superposition of the 4 states from 2.16, all having the same probability. Hence Bob would not be able to deduce from this any information about $|\phi\rangle$, which makes sense since otherwise the signal would travel faster than light.

It would be interesting to see if other states than an EPR singlet could be used for teleportation. In fact, it can be shown that the most effective configuration for teleportation is a state of two maximally entangled particles (i.e they form an EPR singlet). Indeed, we have the following property:

Proposition 2. *Consider a state $|\Upsilon_{23}\rangle$. Then Bob's particle 3 will be related to $|\phi_1\rangle$ by 4 fixed unitary operations if and only if*

$$|\Upsilon_{23}\rangle = \frac{1}{\sqrt{2}} \left(|u_2\rangle |p_2\rangle + |v_2\rangle |q_3\rangle \right), \tag{2.24}$$

where $\{|u\rangle, |v\rangle\}$ and $\{|p\rangle, |q\rangle\}$ are two pairs of orthonormal bases.

Hence 2 and 3 must be maximally entangled. Less entangled states will be less effective at teleportation, either limiting the accuracy of the teleportation or the range of possible $|\phi_1\rangle$ that can be teleported.

It can also be proven that a classical channel of two bits of information is necessary for teleportation. To do so, we study a 4-way coding scheme: B receives two bits of information and one particle of an EPR pair, sending out a particle in state $|\phi\rangle$ and an uncorrelated two bits, while A receives the other particle of the pair and the particle in state $|\phi\rangle$, sending out two bits of information. This is one way to transmit physically the 2 bits of information.

We can thus build a setup where B and A perform a 4-way coding, but in between the particle in state $|\phi\rangle$ gets intercepted and teleported by A' and B'. Suppose now that A' and B' use a channel of capacity C<2 bits, but is still capable of teleporting $|\phi\rangle$, hence also the 2 bit message that B sends to A. If B' were to guess the message superluminally, his probability 2^{-C} of guessing right would be bigger than $\frac{1}{4}$, resulting in a probability bigger than $\frac{1}{4}$ of sending the message superluminally from B to A. Hence there exist two distinct two-bit messages r and s, such that $P(r \mid s) < \frac{1}{4}$, probability of receiving superluminally r if s was sent, and $P(r \mid r) > \frac{1}{4}$, probability of receiving superluminally r if r was sent. It would thus be possible to reliably send messages superluminally, which thus, by contradiction, supposes that $C \geq 2$. By the same argument, teleportation of an N-state particle needs a classical channel of $2\log_2(N)$ bits.

After studying the EPR Paradox and Quantum Teleportation, in order to understand ER=EPR, we now need to direct our attention to ER Bridges.

2.4 Einstein equation, Penrose diagrams and the Schwarzschild black hole

We now shift our focus to general relativity. We will see in section 3 that general relativity and quantum mechanics share a connection, in the form of wormholes. wormholes, or ER bridges, are a theoretical result of general relativity. We first start with the fundamental part of general relativity, Einstein's equation. From there, we will obtain Schwarzschild's solution to Einstein's equation, which directly leads to the Schwarzschild black hole and to the easiest wormhole to describe. This section is based on [8].

Einstein's equation

Einstein's equation determines the local spacetime geometry, given by the metric $g_{\mu\nu}$ [9]. It can be derivated from the Hilbert action, given by:

$$S_H = \int \sqrt{-g} R \, d^n x, \tag{2.25}$$

where $g = |g_{\mu\nu}|$, and R is the Ricci constant [10]. We now need to introduce a few objects, presented in chapters 2 and 3 of [8]. The derivation of Einstein's equation will be based on chapter 4. We use tensorial summation notation, also known as Einstein notation: the expression is ummed over all indices that appear as both subscripts and superscripts.

We will use the Christoffel symbol

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu} \right), \tag{2.26}$$

which enables us to define the covariant derivative of a tensor $T^{\mu_1 \ldots \mu_k}_{\nu_1 \ldots \nu_l}$ as

$$\begin{aligned} \nabla_\mu T^{\mu_1 \ldots \mu_k}{}_{\nu_1 \ldots \nu_l} &= \partial_\mu T^{\mu_1 \ldots \mu_k}{}_{\nu_1 \ldots \nu_l} \\ &+ \sum_{i=1}^k \Gamma^{\mu_i}_{\sigma\lambda} T^{\mu_1 \ldots \mu_{i-1} \lambda \mu_{i+1} \ldots \mu_k}{}_{\nu_1 \ldots \nu_l} \\ &+ \sum_{i=1}^l \Gamma^{\lambda_i}_{\sigma\nu_i} T^{\mu_1 \ldots \mu_k}{}_{\nu_1 \ldots \nu_{i-1} \lambda \nu_{i+1} \ldots \nu_l}, \end{aligned} \tag{2.27}$$

and the Riemann tensor

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}. \tag{2.28}$$

From this, we can get the Ricci tensor:

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} \tag{2.29}$$

and finally the Ricci constant given by

$$R = g^{\mu\nu} R_{\mu\nu}. \tag{2.30}$$

We will also make use of Stoke's theorem:

Theorem 3 (Stokes Theorem)**.** *Let M be an n-dimensional region with boundary δM. Suppose we use coordinates x^i in M, which has metric g_{ij}, and coordinates y^i on δM which has metric γ_{ij}, and suppose that n^μ is the unit normal to δM. Then, for a vector V^μ:*

$$\int_M d^n x \sqrt{|g|} \nabla_\mu V^\mu = \int_{\delta M} d^{n-1} y \sqrt{|\gamma|} n_\mu V^\mu \tag{2.31}$$

One can find, using this result and following the derivation from [8], that

$$\delta S_H = \int d^n x \sqrt{-g} \left[R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right] \delta g^{\mu\nu}. \tag{2.32}$$

Since the action is given by

$$S = \int \sum_i \left(\frac{\delta S}{\delta \phi^i} \delta \phi^i \right) d^n x, \tag{2.33}$$

hence

$$\frac{1}{\sqrt{-g}} \frac{\delta S_H}{\delta g_{\mu\nu}} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{2.34}$$

The Hilbert action is the action exerted by gravity alone. To get the total action, we need to add the action due to matter fields as well. The total action is given by:

$$S = \frac{1}{16\pi G} S_H + S_M, \tag{2.35}$$

S_M being the action due to matter fields. By the principle of least action, we get that:

$$\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} = \frac{1}{16\pi G} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}} = 0. \tag{2.36}$$

If we define the energy-momentum tensor as

$$T_{\mu\nu} = -2 \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}}, \tag{2.37}$$

we then get the Einstein equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{2.38}$$

Before continuing, let us introduce Penrose diagrams.

Penrose Diagrams

Like a Minkowski diagram, which represents flat spacetime in special relativity, a Penrose diagram allows us to represent causality in an understandable way. But in contrast to Minkowski diagrams, Penrose diagrams are conformally equivalent to the actual metric of spacetime, and allow us to represent all of spacetime on a finite diagram.

Figure 1 is a good example of a Penrose diagram. Every point on the diagram is a 2-sphere, and each hyperbola that is drawn represents a curve of either constant time (if horizontal) or constant space coordinate (if vertical). i^0 represents spatial infinity (note that the diagram is left-right symmetric), i^+ is the future timelike infinity and i^- is the past timelike infinity.

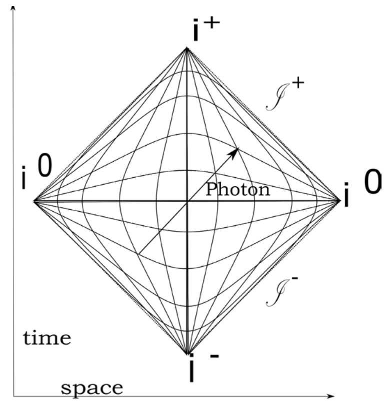

Figure 1. Penrose diagram of spacetime

We say that $\mathscr{I}^+$ is the future null infinity and $\mathscr{I}^-$ the past null infinity. On such a diagram, light moves at a 45° angle with the time and space axes (as shown by the photon), hence, as all events B that another event A can influence lie within the light cone of A, all such B lie within a 45° angle with the time axis from A. Also, all timelike curves start at i^- and end at i^+, all spacelike curves start and end at each i^0, and lightlike curves start on $\mathscr{I}^-$ and end on $\mathscr{I}^+$.

Penrose diagrams are useful for representing black holes and wormholes. We will now work out one of the solutions of Einstein's equation which describes a black hole.

Schwarzschild Solution

The Schwarzschild solution is one solution to (2.38), and the earliest example of a black hole and, as a matter of fact, of a wormhole. We will derive it here as an exercise.

In GR, the unique static spherically symmetric vacuum solution to (2.38) is the Schwarzschild metric [11][12]. A breakdown of all the terms:

- "static" means that the components of $g_{\mu\nu}$ do not depend on t, and that there are no cross terms of type $dtdx^i$ or $dx^i dt$ in the metric.
- "spherically symmetric" means that, in spherical coordinates (t, r, θ, ϕ), the metric depends on

$$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2. \tag{2.39}$$

- "vacuum solution" means that 2.38 reduces to

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0 \Rightarrow R_{\mu\nu} = 0. \tag{2.40}$$

We can thus try to construct such a metric. It will be of the type

$$ds^2 = -e^{2\alpha(r)} dt^2 + e^{2\beta(r)} dr^2 + e^{2\gamma(r)} r^2 d\Omega^2. \tag{2.41}$$

Performing a change of variables

$$\bar{r} = e^{\gamma(r)} r$$
$$\text{with } d\bar{r} = e^\gamma dr + e^\gamma r d\gamma = \left(1 + r\frac{d\gamma}{dr}\right) e^\gamma dr, \tag{2.42}$$

we get

$$ds^2 = -e^{2\alpha(r)} dt^2 + \left(1 + r\frac{d\gamma}{dr}\right)^{-2} e^{2\beta(r) - 2\gamma(r)} d\bar{r}^2 + \bar{r}^2 d\Omega^2, \tag{2.43}$$

and we can now relabel $\bar{r} \to r$ and

$$\left(1 + r\frac{d\gamma}{dr}\right)^{-2} e^{2\beta(r) - 2\gamma(r)} \to e^{2\beta}, \tag{2.44}$$

giving the metric

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + r^2 d\Omega^2. \tag{2.45}$$

Let us now use Einstein's equation to solve for α, β. The Christoffel symbols are given by:

$$\begin{array}{ccc} \Gamma^t_{tr} = \partial_r \alpha & \Gamma^r_{tt} = e^{2(\alpha-\beta)}\partial_r \alpha & \Gamma^r_{rr} = \partial_r \beta \\ \Gamma^\theta_{r\theta} = \frac{1}{r} & \Gamma^r_{\theta\theta} = -re^{-2\beta} & \Gamma^\phi_{r\phi} = \frac{1}{r} \\ \Gamma^r_{\phi\phi} = -re^{-2\beta}\sin^2\theta & \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta & \Gamma^\phi_{\theta\phi} = \frac{\cos\theta}{\sin\theta} \end{array} \tag{2.46}$$

bearing in mind that all other elements are either 0 or related by symmetry to those indicated. We thus get the Riemann tensor elements:

$$\begin{aligned} R^t{}_{rtr} &= \partial_r\alpha\partial_r\beta - \partial_r^2\alpha - (\partial_r\alpha)^2 \\ R^t{}_{\theta t\theta} &= -re^{-2\beta}\partial_r\alpha \\ R^t{}_{\phi t\phi} &= -re^{-2\beta}\sin^2\theta\partial_r\alpha \\ R^r{}_{\theta r\theta} &= re^{-2\beta}\partial_r\beta \\ R^r{}_{\phi r\phi} &= re^{-2\beta}\sin^2\theta\partial_r\beta \\ R^\theta{}_{\phi\theta\phi} &= \left(1 - e^{-2\beta}\right)\sin^2\theta \end{aligned} \tag{2.47}$$

giving us the elements of the Ricci tensor:

$$\begin{aligned} R_{tt} &= e^{2(\alpha-\beta)}\left[\frac{2}{r}\partial_r\alpha - \partial_r\alpha\partial_r\beta + \partial_r^2\alpha + (\partial_r\alpha)^2\right] \\ R_{rr} &= \frac{2}{r}\partial_r\beta + \partial_r\alpha\partial_r\beta - \partial_r^2\alpha - (\partial_r\alpha)^2 \\ R_{\theta\theta} &= e^{-2\beta}\left[r\left(\partial_r\beta - \partial_r\alpha\right) - 1\right] + 1 \\ R_{\phi\phi} &= \sin^2\theta R_{\theta\theta}. \end{aligned} \tag{2.48}$$

Since $R_{\mu\nu} = 0$, and since R_{tt} and R_{rr} vanish independently, we can set:

$$0 = e^{2(\alpha-\beta)}R_{tt} + R_{rr} = \frac{2}{r}\left(\partial_r\alpha + \partial_r\beta\right)$$

giving us $\alpha = -\beta + c,\ c \in \mathbb{R}$. Rescaling $t \to e^{-c}t$ gives

$$\alpha = -\beta. \tag{2.49}$$

We also set $R_{\theta\theta} = 0$, yielding

$$\begin{aligned} e^{2\alpha}\left(2r\partial_r\alpha + 1\right) &= 1 \\ \Rightarrow \partial_r\left(re^{2\alpha}\right) &= 1 \\ \Rightarrow e^{2\alpha} &= 1 - \frac{R_S}{r}, \end{aligned}$$

where R_S is a constant called the Schwarzschild radius. Since we have $g_{tt} = -e^{2\alpha} = -\left(1 - \frac{R_S}{r}\right)$ and in the weak-field limit it satisfies $g_{tt} = -\left(1 - \frac{2GM}{r}\right)$ around a point mass, we obtain that

$$R_S = 2GM, \tag{2.50}$$

and

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 + r^2d\Omega^2 \tag{2.51}$$

is the Scharzschild metric. We can think of (2.50) as the definition of M.

The metric diverges for $r = 0$ and $r = 2GM$. Since $R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{48G^2M^2}{r^6}$ diverges for $r = 0$, it is a singularity of the metric.

The behavior of matter outside $r = 2GM$ can be nicely described, but the interesting part comes from analyzing the Schwarzschild solution at $r < 2GM$.

Schwarzschild black holes

Let us now, in an attempt to understand causality in the Schwarzschild metric, study radial null curves, for which θ and ϕ are constant. By definition, $ds^2 = 0$ on null/lightlike curves. As a quick reminder we have $ds^2 < 0$ for timelike curves and $ds^2 > 0$ for spacelike curves:

$$ds^2 = 0 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2,$$

implying

$$\frac{dt}{dr} = \pm\left(1 - \frac{2GM}{r}\right)^{-1}, \tag{2.52}$$

which is the slope of the light cones on a spacetime diagram of the $t - r$ plane. We have $\lim\limits_{r\to+\infty}\frac{dt}{dr} = \pm 1$, and $\lim\limits_{r\to 2GM}\frac{dt}{dr} = \pm\infty$. It thus seems that the light rays never reach $r = 2GM$,but that is just an illusion caused by our coordinate system (hence an outside observer will never see the light rays reach that point). In order to see what really happens, we need to find a more suitable coordinate system. First, let us take $t \to \pm r^* + \text{const}$, where $r^* = r + 2GM\ln\left(\frac{r}{2GM} - 1\right)$. Our metric thus becomes well behaved at $r = 2GM$ (meaning that it is not a singularity after all). Now, define

$$\begin{aligned} v &= t + r^* \\ u &= t - r^* \end{aligned} \tag{2.53}$$

and use the coordinate system (v, r, θ, ϕ), known as the Eddington-Finkelstein coordinates. The metric now becomes:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dv^2 + (dv\,dr + dr\,dv) + r^2d\Omega^2 \tag{2.54}$$

and the determinant of the metric is $g = -r^4\sin^2\theta$, well-behaved at $r = 2GM$. The condition for radial null-curves is solved by:

$$\frac{dv}{dr} = \begin{cases} 0, & \text{(infalling)} \\ 2\left(1 - \frac{2GM}{r}\right)^{-1}. & \text{(outgoing)} \end{cases} \tag{2.55}$$

This result is interesting. The light cones remain well-behaved, but for $r < 2GM$, they close up ($\frac{dv}{dr} < 0$ for outgoing curves for $r < 2GM$), hence all future-directed paths are in the direction of decreasing r. Hence, $r = 2GM$ is a point of no return: this is the event horizon. Since nothing can escape it, the whole region lying within $r < 2GM$ is called a black hole.

We can extend the Schwarzschild solution even further to include even more regions.

Indeed, in the (v, r) coordinates, we can cross the event horizon on future-directed, but not on past-directed paths. If we now choose to replace v by u from (2.53), our metric becomes

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)du^2 - (du\,dr + dr\,du) + r^2d\Omega^2 \tag{2.56}$$

for which we get

$$\frac{du}{dr} = \begin{cases} -2\left(1 - \frac{2GM}{r}\right)^{-1}, & \text{(infalling)} \\ 0. & \text{(outgoing)} \end{cases} \tag{2.57}$$

The event horizon can only be crossed on past-directed paths. Where the coordinatees (v, r, θ, ϕ) extended spacetime to the future, the coordinates (u, r, θ, ϕ) extended it to the past. We call that region of spacetime a white hole (a region from which things can escape to us, while we cannot get there).

Following spacelike geodesics uncovers even another region. Using the coordinates

$$\begin{aligned} v' &= e^{v/4GM} \\ u' &= e^{-u/4GM} \end{aligned} \tag{2.58}$$

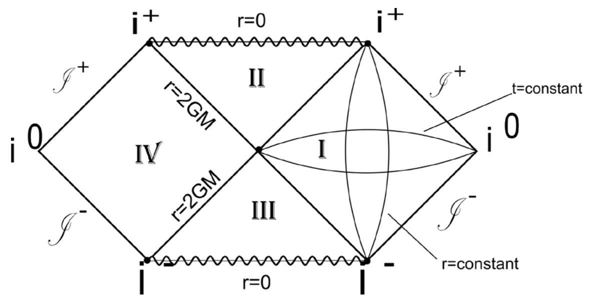

Figure 2. Penrose diagram of the Schwarzschild metric. Region I is our asymptotically flat region outside $r = 2GM$, Region II is the black hole, Region III the white hole and Region IV the other asymptotically flat region. $r = 0$ represents the singularity. Note that only spacelike curves connect regions I and IV, and that all timelike curves on II end at $r = 0$ and on III start at $r = 0$.

which can be expressed as

$$\begin{aligned} v' &= \left(\frac{r}{2GM} - 1\right)^{\frac{1}{2}} e^{(r+t)/4GM} \\ u' &= -\left(\frac{r}{2GM} - 1\right)^{\frac{1}{2}} e^{(r-t)/4GM}, \end{aligned} \tag{2.59}$$

we can create the new set of coordinates (T, R, θ, ϕ), where T and R are defined as

$$\begin{aligned} T &= \frac{1}{2}(v' + u') = \left(\frac{r}{2GM} - 1\right)^{\frac{1}{2}} e^{r/4GM} \sinh\left(\frac{t}{4GM}\right) \\ R &= \frac{1}{2}(v' - u') = \left(\frac{r}{2GM} - 1\right)^{\frac{1}{2}} e^{r/4GM} \cosh\left(\frac{t}{4GM}\right). \end{aligned} \tag{2.60}$$

Thus, using $T^2 - R^2 = \left(1 - \frac{r}{2GM}\right) e^{r/2GM}$, the metric becomes

$$ds^2 = \frac{32G^3M^3}{r} e^{-r/2GM}(-dT^2 + dR^2) + r^2 d\Omega^2. \tag{2.61}$$

With these coordinates, the metric covers all of spacetime for which $-\infty \leq R \leq +\infty, T^2 < R^2 + 1$. We can thus uncover a fourth region: another asymptotically flat spacetime which can only be reached from the other regions by spacelike curves. That is we can not reach it, nor can anything from it reach us. We can think of it as a region connected to the region $r > 2GM$ by an ER bridge. The entirety of spacetime covered by the Schwarzschild metric can be represented by Figure 2.

We can think of the non-traversable ER-bridge, if we were to slice the diagram into spacelike slices of constant time, as the two regions I and IV reaching for each other, join together through a wormhole for a while, then disconnect. The wormhole closes too quickly for anyone to traverse it.

Now that we have seen both the EPR Paradox and ER Bridges, we can start reviewing ER=EPR.

3 ER=EPR, wormhole teleportation and wormhole dynamics.

We can now finally make a connection between general relativity and quantum mechanics, by connecting ER Bridges and the EPR Paradox through the statement ER=EPR. It is a quite recent hypothesis[3], with certain implications on the feasibility of teleportation using wormholes. We will thus review both concepts here, before diving into our own deductions and contributions.

3.1 ER=EPR

Maldacena and Susskind present in their paper "Cool horizons for entangled black holes" [3] a conjecture on Einstein Rosen bridges (ER) and quantum entanglement (EPR), claiming that both phenomena are the same.

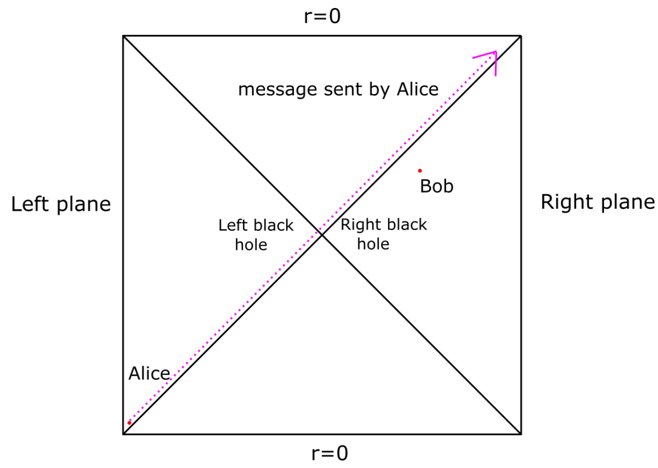

Figure 3. Penrose diagram of the situation described: Alice sends a message through her black hole, which can only be intercepted by Bob if he enters his. If Alice sends it early enough, and her message is a deadly set of quanta, then this situation represents a firewall from Bob's perspective.

Firstly, they present different ER bridges, like the AdS black hole or the Schwarzschild black hole as presented here in section 2.4. It is shown that the ER bridge onnecting each of the two black holes in each case can be described at $t = 0$ as an entangled state $|\psi\rangle = \sum_n e^{-\beta \frac{E_n}{2}} |n, n\rangle$, and that, if we consider the time evolution of $|\psi\rangle$ to be

$$|\psi_t\rangle = \sum_n e^{-\beta \frac{E_n}{2}} e^{-2iE_n t} |n, n\rangle, \tag{3.1}$$

each $|\psi_t\rangle$ can also represent a different state. They thus claim that black holes represented in such a way do not have firewalls, and that we can instead consider Figure 3.

They also describe a way of creating an ER bridge by using quantum entanglement, and how the wormhole geometry varies with time.

After that, the authors present their main claim: that an ER bridge and an EPR entanglement are the same thing (hence ER=EPR). They base this on the numerous similarities between the two phenomena (i.e. that both phenomena do not violate locality and that both forms of connection between two shares either need a pre-existing connection or direct contact of the two connected shares to be created, that is they cannot be created by local operations or classical communication). They therefore claim that every instance of an ER bridge is also an instance of quantum entanglement, and vice versa. The paper also explores the possibility of Einstein-Rosen bridges which, instead of connecting two black holes, connect a whole set of particles together. That way, every two subsystems are maximally entangled and thus connected by the ER bridge. They also study how such a system would behave, and how Hawking radiation is such a cloud entangled with a black hole.

3.2 Using wormholes for teleportation

Assuming ER=EPR, it should be possible to teleport information through a wormhole exactly like one can teleport information using an entangled pair of particles. This idea is presented in the paper "Teleportation through the wormhole" [5] , of which we present now a summary, as well as some further implications of ER=EPR:

Some prerequisites

A black hole of entropy N will be represented by N qubits, and the Hamiltonian by $H = \sum_{i,j} h_{ij}$, h depending on the Pauli operators. The time-evolution $U(t) = e^{-iHt}$ is used to express precursor operators $U^{\dagger}(t)\, A U(t)$, where A is an operator that adds or subtracts particles to the system, which represent a system to which A is applied at time t. Note that $U(t)$ and $U^{\dagger}(t)$ act on a different number of qubits.

As such a system needs a time $t_* = \log N$ to scramble, the corresponding time-evolution operator for an N qubit system until scrambling is denoted by $V_N = e^{-iHt_*}$. The complexity of V_N $N \log N$, hence that of $V^{\dagger} W V$ for a simple two qubit operator W is, by the switchback effect, N.

The actual protocol

Let us now assume that Alice and Bob each have one throat of the wormhole, each portion labeled **A** and **B** respectively, which is modeled by a system of $2N$ maximally entangled qubits, such that its entropy is N. Suppose that Alice wants to teleport a system of n particles in the state $|\phi\rangle$denoted by **T**. We need to take into account that $n \leq N$ and that Alice needs to send a classical message of size $2n$ for this to work. We consider the example case $n = 1$.

We assume that the initial state of the system is the thermofield-double state

$$|\mathrm{TFD}\rangle = \sum_I |I\rangle_A |I\rangle_B \,, \tag{3.2}$$

where $|I\rangle_{A,B}$ is the complete set of states in the chosen bases of **A** and **B**'s systems. If k is a complete set of states in the Hilbert space of **T**, i.e. $|0\rangle$, $|1\rangle$, then **T**'s state is

$$|\phi\rangle_T = \sum_k \phi(k) |k\rangle_T \,. \tag{3.3}$$

The initial state of the whole system is thus

$$|\mathrm{initial}\rangle = \sum_{I,k} \phi(k) |k\rangle_T |I\rangle_A |I\rangle_B \,. \tag{3.4}$$

Before the teleportation, the qubit of **T** is absorbed by the black hole **A** and scrambled. This is represented by applying the operator V on the initial state:

$$V |\mathrm{initial}\rangle = \sum_{I,k} \phi(k)\, V |kI\rangle_{AT} |I\rangle_B \,. \tag{3.5}$$

Picking now any two qubits from (**AT**), and labeling them as θ, in the state $|\theta\rangle$, and all the other particles from that system as α, our system is thus in the state:

$$\sum_{I,k,\theta,\alpha} \phi(k) |\theta, \alpha\rangle \langle\theta, \alpha| V |kI\rangle_{AT} |I\rangle_B \,. \tag{3.6}$$

Defining now $V_{kI}^{\theta,\alpha} \equiv \langle\theta, \alpha| V |kI\rangle$, (3.6) becomes

$$\sum_{I,k,\theta,\alpha} \phi(k)\, V_{kI}^{\theta,\alpha} |I\rangle_B |\theta, \alpha\rangle \,. \tag{3.7}$$

Alice then measures θ, obtaining a specific result which she then sends to Bob, who then acts on **B** with a unitary operator Z^{θ}, which bijectively depends on θ, obtaining $|\phi\rangle$ as a result. The paper studies how to choose Z^{θ} in order to minimize the time between sending and receiving $|\phi\rangle$, but here we will just focus on the protocol. We therefore jump directly to the necessary operator Z^{θ}, which minimizes the time complexity:

$$Z^{\theta} = U^{\dagger}(t_*) \left(\sum_{\gamma} V_{I,j}^{\theta,\gamma} V_{\gamma\beta}^{\dagger} \right) U(t_*) \,. \tag{3.8}$$

3.3 Consequences of ER=EPR, wormhole dynamics, electromagnetic wave equation and photon energy-momentum tensor

Even if a wormhole teleportation protocol exists, it is still necessary for a wormhole connecting two particles as a quantum entanglement (and thus serving as a bridge for information from one particle to affect the other as claimed in [5]) to either be traversable or to allow information in its interior to affect its exterior. We prove this qualitatively:

Claim 1. A wormhole connecting two particles that are entangled is either traversable or lets information permeate, supposing ER=EPR holds.

Proof. If we suppose that every quantum entanglement is an instance of a wormhole, and that the influence of the measurement of one of the connected systems on the value taken by the other is in fact just an information transfer through that wormhole connecting both systems, then this would either mean that such a wormhole is traversable, or that the information inside a wormhole can have observable influences on the outside of the wormhole, in order for the information to affect the second particle. ∎

We explore here and in Appendices A and B the possibility that the incoming information changes the metric if ER=EPR is valid, which raises the question about the validity of the no-hair development. To this end, we will study the particular case of an incoming photon moving towards a Schwarzschild black hole in the original (t, r, θ, ϕ) coordinate system, calculating the energy-momentum produced by a particular solution to the electromagnetic wave-equation in curved spacetime.

Finding the energy-momentum tensor

By Appendices A and B, in which we determine how to calculate the energy momentum tensor of our metric from the Faraday tensor for Appendix A and how to obtain the Faraday tensor by solving the electromagnetic wave equation in curved spacetime for Apendix B, we can determine the Faraday tensor from the metric and the electromagnetic potential A_μ:

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \Gamma^{\lambda}_{\mu\nu} A_\lambda - \partial_\nu A_\mu + \Gamma^{\lambda}_{\nu\mu} A_\lambda \tag{3.9}$$

and since $A_\mu = g_{\mu\lambda} A^\lambda$, we can determine that all elements of $F_{\mu\nu}$ are 0 except for:

$$\begin{aligned} F_{tr} = -F_{rt} &= \partial_t A_r - \Gamma^t_{tr} A_t - \partial_r A_t + \Gamma^t_{rt} A_t = \partial_t A_r - \partial_r A_t \\ &= \partial_t \left(g_{rr} A^r\right) - \partial_r \left(g_{tt} A^t\right) \\ &= g_{rr} \partial_t A^r - \partial_r g_{tt} A^t - g_{tt} \partial_r A^t. \end{aligned} \tag{3.10}$$

After solving the wave equation, we get a spherically symmetric wave. Indeed, using (**??**), setting

$$f_3 = \left(\frac{\left(\partial_r f_1 f_2 + f_1 \partial_r f_2\right)(r - 2GM) + 2 f_2 f_1 (r - 2GM) - \frac{8G^2M^2\omega^2}{r^3}}{\left(f_2 f_1 (r - 2GM)^2 + \frac{4G^2M^2\omega^2}{r^2}\right)^2} + \frac{i\omega}{f_2 + \frac{4G^2M^2\omega^2}{f_1 (r-2GM)^2 r^2}} \right),$$

we get

$$\begin{aligned} F_{tr} = \int \Bigg[&- \frac{i\omega}{f_2 (r - 2GM) + \frac{4G^2M^2\omega^2}{f_1 (r-2GM) r^3}} \\ &+ \frac{i\omega 4G^2M^2}{f_2 f_1 r^2 (r - 2GM)^2 + 4G^2M^2\omega^2} \\ &+ i\omega 2GM \left(1 - \frac{2GM}{r}\right) f_3 \Bigg] \tilde{J}^r e^{i(kr - \omega t)} \mathrm{d}k \, \mathrm{d}\omega. \end{aligned} \tag{3.11}$$

We can see that F_{tr} is non-zero.

From this, we get, with $F^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}$, the energy-momentum tensor generated by the particular photon producing the electromagnetic potential **??** in the Schwarzschild metric:

$$T_{\mu\nu} = \begin{pmatrix} -\frac{3}{2}\left(1 - \frac{2GM}{r}\right) F_{tr}^2 & 0 & 0 & 0 \\ 0 & \frac{3}{2} \frac{1}{\left(1 - \frac{2GM}{r}\right)} F_{tr}^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2} r^2 F_{tr}^2 & 0 \\ 0 & 0 & 0 & \frac{1}{2} r^2 \sin^2 \theta F_{tr}^2 \end{pmatrix}, \tag{3.12}$$

which has elements that are non-zero. Therefore, we can conclude that the incoming photon indeed has an effect on the metric.

We must stress that this result is interesting, as this seems to help substantiate the claim that an object entering a wormhole changes its metric. The case with a photon is very particular as it does not directly contradict the no-hair theorem, which states that the only parameters affecting the metric of a black hole are its mass, angular momentum and charge. As the photon carries angular momentum, this should be what affects the metric. More work is needed to see if, for photons or massive objects, the change in the metric can be directly related to the mass, charge and angular momentum of the incoming object, or if there is other information affecting it. We could even go a bit further and imagine that the change in the metric is uniquely determined by the information of the infalling object, which would lead to another protocol for wormhole teleportation than that of [5]. And even if the metric remains unaffected by the information, with Claim 1, we have established that certain wormholes are either traversable or let information permeate if ER=EPR holds, meaning that that can also be used for teleporting information (say the spin of a particle).

Further Developments

Although we did not solve Einstein's equation in order to see how exactly the metric changes, we think that the result might be worth studying. We expect the metric of the wormhole to change uniquely with respect to the information sent in, but to do this we would need to solve Einstein's equation using (3.12) as the energy-momentum tensor to determine the new metric. We also want to point out that we have used here the Schwarzschild metric in the original coordinate system, which only properly describe the metric outside the black hole. This is probably useful to determine how a photon inside might affect the metric outside, but we also think that studying this in the Kruskal-Szekeres coordinates is probably better. Additionally, the Schwarzschild black hole is not a good contender for a wormhole connecting two entangled particles, hence we would need to study different types of black holes/wormholes for better results.

It is also worth noting that a change in the metric is not the only possible explanation for information permeability. black hole complementarity might be a good candidate, or maybe even Hawking radiation. Traversable wormholes would also let information permeate for obvious reasons. Finally, we must also consider the possibility that ER=EPR does not hold.

4 Conclusion

We have reviewed the basics of quantum teleportation and of wormholes, and subsequently one of the implications of the claim ER=EPR, that is that a wormhole and a quantum entanglement are the same object, namely how, analogously to quantum teleportation, wormholes could be used to teleport information. We have also determined that certain wormholes have to be either traversable or at the very least let the information stored inside somehow affect the outside in an observable way. We have advanced the claim that the information might change the metric at the receiving end of the wormhole, and have started to explore this path. Unfortunately, we were only able to obtain the energy-momentum tensor, without being able to either solve Einstein's equation or obtaining it in the Kruskal-Szekeres coordinate system.

We have only started exploring one of the possible ways information swallowed by one throat of a wormhole might affect the other end in an observable way. Other ways that could be explored would be black hole complementarity [13] for example. Several papers, such as [14] and [15] have established theoretical frameworks in which traversable wormholes are possible, and it would be interesting to examine if such a wormhole might be a good candidate for the usual quantum entanglement.

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