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Review Article

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Conflicts with de Sitter Vacua in Superstring Theory

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Abstract

Models of our universe lack consistency at different energy scales, so we require a theory with ultraviolet (UV) completion such as string theory. A suitable candidate to model our universe in this framework is de Sitter space, a spacetime which expands and has positive curvature. When describing the expansion of this space, however, one computes the wrong sign for the cosmological constant that would not allow for an expanding universe. This motivates one to consider corrections from a quantum theory to reproduce the correct positive sign for the cosmological constant. The conditions that cause this incorrect sign are known as the *swampland criteria*, and prevent de Sitter space from being realized in a consistent manner at different energy scales. We look at a framework to avoid the swampland restriction in a UV-complete theory by considering de Sitter space resulting from compactifications of type IIB superstring theory. In particular, we demonstrate that the definitions of particles in an expanding UV-incomplete theory leads to inconsistencies in the definition of the de Sitter vacuum states. Furthermore, we review previous attempts to prevent these inconsistencies by constructing coherent states that expand and have the desired de Sitter isometries over supersymmetric Minkowski space. These states add quantum corrections to the metric operator, resulting in the cosmological constant carrying the correct sign. Therefore, the de Sitter space can be used in a UV-complete theory to model our universe.

Introduction

It is of great interest in theoretical physics to develop a complete theory of our universe that remains consistent across all energy scales. Current theories exist at very small energy scales (and thus large length scales), like general relativity, or at very high energy scales (and small length scales), such as quantum field theory (QFT). Typically, a theory is defined up to a certain energy scale — *a cutoff* — and above it the equations of motion do not apply. Extending a theory beyond this cutoff to make it well-defined at arbitrarily high energy scales is termed an *ultraviolet (UV) completion*. In a UV-complete theory, the features at high energy scales imply those at low energy scales, and vice versa. In this sense, there is a mixing between the degrees of freedom of theories at high energy scales (ultraviolet) and low energy scales (infrared), known as *UV/IR mixing*. A theory consistent across all energy scales must therefore be UV-complete and exhibit UV/IR mixing. Since such a theory is local in nature, it requires a local quantum field theory description[1]. So far, two theories that have accurately predicted natural phenomena are QFT and general relativity. However, neither exhibits UV/IR mixing, and since they are defined for different energy scales, they do not share the same features; hence, they do not form a complete theory. The attempt to reconcile a quantum description of particles with a classical theory of gravity is known as the problem of *quantum gravity*.

One candidate theory of quantum gravity is string theory, a UV-complete field theory. Although string theory (hence superstring theory as well) is self-consistent, meaning alternate computation methods yield the same results, scientists are still unable to construct a model that reproduces the required properties of nature. Furthermore, many theories that do reproduce these properties, such as QFT or general relativity, reveal flaws under closer inspection, including divergent observables and geometries. In this paper, we are concerned with the *cosmological constant*. This constant determines the universe's rate of expansion: a positive sign corresponds to an expanding universe while a negative sign (or vanishing constant) does not. Under a theory with an expanding spacetime, classical computations do not yield the right sign due to conditions known as the *swampland criteria*. This motivates the introduction of quantum corrections from a quantum theory to bypass these classical constraints, allowing theories to fall outside of the *swampland*. By using type IIB superstring theory, we construct a UV-complete model of our current universe — which contains IR/UV mixing — and avoid the divergence issues of other theories.

To begin, we live in a 3+1D universe (3 spatial dimensions, 1 time dimension) where the curvature of spacetime is nearly flat. Type IIB string theory, however, is consistent in 9+1D, which is six more dimensions than our universe. How then can we recover our lower-dimensional universe from this higher-dimensional theory? The process is called *compactification*[*]. The extra six dimensions form a *compact internal space* that, roughly speaking, we take to be small. This process is similar to the construction of a Riemann sphere in complex analysis, whereby a point from infinity is brought to the complex plane to form the compact Riemann sphere. A visualization of the compactification of a torus is shown in Figure 1.

In our case, we split the 9+1D theory into a 3+1D piece (our universe) and a 6D piece (the internal space), and recover our universe via compactification of the internal space. Now that we can recover our universe from a higher dimensional one, the next step is to select which universe model to use. One positively curved and expanding candidate model is 4D de Sitter space dS_4, a vacuum solution to the Einstein field equations. In it, the vacuum states (not to be confused with vacuum *solutions*) share the symmetries of the space and allow for the computation of observables one would measure in a laboratory. One such observable is the *dark energy*, directly related to the spacetime's cosmological constant. As previously mentioned, this constant is crucial as it determines the universe's expansion rate, with a positive constant expected for expanding space.

To construct de Sitter vacua within a UV-complete theory, one method is

[*]Compactification is a process in general topology where we take a topological space or manifold, usually one of the extra/internal higher dimensions of the theory, and make it into a compact space[2]. The physics definition extends to taking this compact space to vanish in the limit that the parameter which modulates its size vanishes (such as taking the radius of an n-sphere S^n to vanish: $R \to 0$).

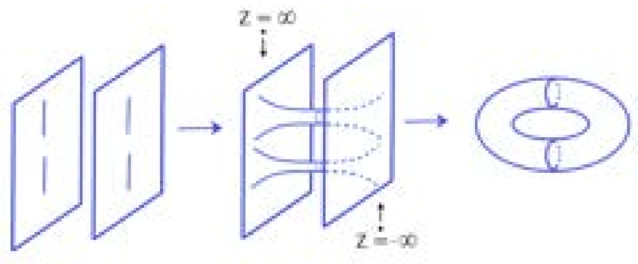

Figure 1. Constructing a torus via compactification. On the left are two parallel 2-dimensional planes with two defined branch cuts over each plane. We connect the branch cuts to get the topology in the middle. Thereafter, we compactify both planes by bringing in points from infinity. The resulting shape is topologically equivalent (diffeomorphic) to a torus.

the KKLT scenario[3], in which a metastable de Sitter state is constructed by uplifting an AdS state via some branes[†] in the presence of a warped geometry in type II string theory. However, it has been shown that this scenario cannot have a well-defined effective field theory as it meets the *swampland criteria*[5], and hence cannot be realized as a consistent theory of gravity. Additionally, in the KKLT regime, the de Sitter conjecture adds no new information in the weak coupling regions where vacua like KKLT are claimed to lie and is violated by the Higgs potential[6]. Another attempt is the Bunch-Davies vacuum, described via a Fourier decomposition of modes over a static patch of de Sitter space[7]. However, because these modes diverge in amplitude at the boundary of the patch, one cannot control the ground state, which may evolve into an excited state. Finally, through compactifications of supergravity, one can construct a de Sitter space (and hence state) subject to certain *classical conditions*[8], under which the components of the metric on the space diverge after compactification. This means that there cannot exist consistent vacuum solutions (i.e. de Sitter space) from compactifications of string theory, leading again to the so-called *swampland* scenario.

As explained, although classically these solutions are forbidden, we can use a quantum theory to obtain quantum corrections that will bypass these conditions and let de Sitter space occur outside the swampland region. We can then obtain a consistent theory of our expanding universe, using de Sitter space, that admits the correct sign for the cosmological constant. Instead of examining vacuum states in expanding geometries like de Sitter space, we instead turn to *Glauber-Sudarshan* states[9]. These generalized coherent states contain all the degrees of freedom of the fields present in string theory and share the isometries of de Sitter space over supersymmetric Minkowski space. Although supersymmetric Minkowski space is flat and does not expand, Glauber-Sudarshan states are expanding and possess the required de Sitter isometries, meaning that the combination remains a relevant candidate to model our universe.

In this paper, we present that in an expanding space — within a UV-incomplete theory — the definitions of particles (and hence their associated vacuum states) become no longer well-defined due to time-dependent frequencies. Because we need a proper definition of vacua states to compute observables in a quantum theory, we require the use of an *alternate* formulation in which the space is taken to be static and the vacua dynamic (which shares the symmetries of the original dynamic space). We look at this alternate formulation by reviewing how coherent states over supersymmetric Minkowski space may be used as a description of vacua in an expanding spacetime. These states add quantum corrections to the space's metric operator, resulting in a positive sign for the cosmological constant. This means that after compactification, the space expands without divergent metric components, making de Sitter space a candidate to model our universe in a UV-complete theory.

[†] In certain topological versions of string theory, *branes* can be viewed as subspaces of Calabi-Yau manifolds[4], a special type of Ricci flat manifold.

de Sitter Vacuum States

We begin by examining how fields behave on expanding geometries in UV-incomplete theories. This is relevant as these fields are exact excitations of the vacuum states in de Sitter space used to calculate the cosmological constant. Specifically, we show that it is impossible to formulate an effective action in an accelerating spacetime because the fields develop time-dependent frequencies. This means we cannot integrate out higher energy modes of fields, and thus *cannot define a theory* at a fixed energy scale. This shows that vacuum solutions of the Einstein field equations in general relativity, including de Sitter space, cannot have consistent descriptions of matter (massive particles) when the solutions are expanding spacetimes. The simplest case we can consider is scalar bosons in an expanding geometry, a review of which is given in Mukhanov, V. & Winitzki, S. (2007)[10], where it is shown that scalar fields develop time-dependent effective mass terms in the Lagrangian, meaning particles have no inherent description of mass. We are interested in cases of higher spin—such as vector bosons or spinor fermion—which could potentially have different statistics or behaviours. To demonstrate the definition of particles breaks down in a dynamic background, we will consider particles with different statistics to scalar bosons which have higher spin: spinor fermions. It is noted that as well as it has different complex structure for its corresponding bundle) see how the definition of particles breaks down.

First, the de Sitter space we are working with is curved and expands to model our universe spacetime. To define fields on equal footing within UV-complete theories over a spacetime manifold, we define them as sections of a *fibre bundle*[11]. A fibre bundle is a non-trivial collection of manifolds that generalizes the notion of product spaces between said manifolds. Since the latter can change at different points, using fibre bundles allows us to define more general field configurations for use in UV-complete theories. A fibre bundle of two manifolds is composed of a *total space* and a *base space*, along with a projection mapping between the two. A fibre bundle can be seen as a prescription of information selection: the base space tells us *how* to select information from the total space. A useful property of fibre bundles is that locally, the space resembles a typical product space. This allows one to work in locally flat spacetime – by defining a local trivialization that uses locally flat charts of $\mathbb{R}^n$. These manifolds are connected by a projective mapping π, which acts on the elements of the total space known as *fibres*. Naturally, the total space is the collection of all the fibres. Furthermore, different fibre bundles are *associated* together if a morphism can be defined between them, such that we can construct one bundle from the other. For example, consider a *spacetime* bundle where the total space is Euclidean space $\mathbb{R}^3$, and the base space is $\mathbb{R}^1$ representing a time dimension. One bundle that can be associated to this spacetime bundle is a *vector* bundle which has the same base space of $\mathbb{R}^1$ as the spacetime bundle, however its fibres are instead vector spaces over the fibres of the spacetime bundle. It is on these vector field fibres that fields (and hence particles) are defined. In the context of type IIB superstring theory in 9+1D, we work with a more general spacetime bundle construction where the base or *external* space is instead all of our 3+1D universe/spacetime $\mathcal{M}^{3,1}$, and the total or *internal* 6D space is M^6. The internal space is compactified to obtain de Sitter geometry.

Now, a few conditions must be met to study *spinors* (elements of complex vector spaces that describe particles of half-integer spin) over an expanding spacetime bundle. An associated complex vector bundle, known as a *spinor bundle*, is required. We define spinors using the spin representation of the Lorentz group, as *sections* of the spinor bundle. Sections are the union of different fibre pieces at different total space points. Being that spacetime is a curved space, a connection is required to study how the derivatives of fields transform in different parts of the space or fibre. This is given by the *covariant derivative*, which acts on these spinors under the following representation:

$$D_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}. \tag{1}$$

Here, ∂_μ is the usual spacetime derivative, ω_μ^{ab} is the *spin connection*, $\gamma_{ab} = \gamma_{[a}\gamma_{b]}$ is the anti-symmetrization of the γ-matrices, and together this represents how the spinor field changes on the space. Similarly to selecting a locally flat patch in a curved manifold, it is useful to work in a locally flat inertial frame of the curved spacetime. This frame is provided by the *frame bundle*: a principal bundle (a fibre bundle possessing group action on a fibre space) that is associated with our spinor bundle. With the frame bundle, we can attach a local frame or coordinate basis to each fibre in the spinor bundle. This allows us to describe the space locally with flat, expanding coordinates given by $tetrad\ coordinates\ e_a^\mu$. These coordinates are sections of the frame bundle that have a Lorentz index μ and a basis index a. Through tetrads, we can write our metric as locally related to the flat Minkowksi metric, bypassing the need for local trivializations (to work in flat space). We can notably define γ-matrices in curved spacetime as:

$$\Gamma^\mu(x) = e_a^\mu(x)\gamma^a, \tag{2}$$

where γ^a are the spatially-constant γ-matrices and e_a^μ is the local tetrad coordinate basis. This provides a notion of spatially-dependent γ-matrices on curved spacetime. The dynamics of the spinors ψ in a curved, expanding spacetime, with the γ-matrices and covariant derivatives, are thus given by the following action:

$$S = \int d^4x\sqrt{-g}\left[\bar{\psi}\Gamma^\mu D_\mu\psi - m\bar{\psi}\psi\right]. \tag{3}$$

Here, $\sqrt{-g}$ is the root determinant of the metric (the negative sign yields a real determinant), which keeps the measure Lorentz-invariant, ψ are the spinor fields, $\bar{\psi} = \psi^\dagger\gamma^0$ are conjugate spinor fields, Γ^μ are the γ-matrices in curved spacetime, D_μ is the covariant derivative associated with the spin connection ω_μ^{ab}, and m is the mass of the spinors. It is sufficient to show that we cannot describe spinors on a curved, expanding spacetime by studying how they fail in a flat, expanding spacetime. To understand the dynamics of these spinor fields, we observe how the expanding metric contributes to the action S on this flat space. Consider the *Friedmann–Lemaître–Robertson–Walker (FLRW)* metric, the simplest metric for a flat expanding universe:

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i \otimes dx^j. \tag{4}$$

In this equation, $a(t)$ is the scale factor that determines how the universe expands, δ_{ij} is the flat metric for the spatial component of the space, $\{dx^i\}$ is the differential form basis of the cotangent space, and $\otimes$ is the tensor product between forms. To simplify calculations, one could relate this to the Minkowski metric under a conformal transformation. Replacing time t with conformal time η (through $dt = a(\eta)d\eta$) results in the following metric:

$$ds^2 = a^2(\eta)(-d\eta^2 + \delta_{ij}dx^i \otimes dx^j). \tag{5}$$

We can compactly write this conformal relation between the FLRW metric $g^{\mu\nu}$ and the Minkowski metric $\eta^{\mu\nu}$ as $g^{\mu\nu} = a^{-2}\eta^{\mu\nu}$. Given that Minkowski space is flat, the spin connection ω_μ^{ab} vanishes, reducing the covariant derivatives D_μ to normal spacetime derivatives ∂_μ. The covariant derivatives therefore act on the spinor fields as $D_0\psi = \partial_0\psi \equiv \dot{\psi}$ and $D_i\psi = \partial_i\psi \equiv \psi_i'$. Here, ψ_i' is written to mimic usual notation for spatial derivatives such as $f'(x) = \partial_x f$, and the index is not to be confused with the ψ_ν vector-spinors. Accordingly, the action becomes:

$$S = \int d^4x\, a^2\left[\bar{\psi}\left(\Gamma^i\psi_i' - \Gamma^0\dot{\psi}\right) + \frac{1}{4}\bar{\psi}\left(\Gamma^i\omega_i^{ab} - \Gamma^0\omega_0^{ab}\right)\gamma_{ab}\psi \right. \\ \left. - ma^2\bar{\psi}\psi\right], \tag{6}$$

where the Dirac conjugate is used for curved spacetime $\bar{\psi} = \psi^\dagger\Gamma^0$. Although we are in flat space, we keep the vanishing spin connection terms to emphasize the structure of the action when we split spatial and temporal indices. Now, to conceive of spinors on an expanding spacetime, we introduce an *auxiliary field* $\chi = a(\eta)\psi$. Taking the spatial and temporal derivatives of χ and relating them to the original spinor results in the following relations:

$$\dot{\psi} = \frac{\dot{\chi}}{a} - \frac{\dot{a}}{a^2}\chi, \quad \psi_i' = \frac{\chi_i'}{a} - \frac{a_i'}{a^2}\chi. \tag{7}$$

Plugging the expressions for $(\psi, \dot{\psi}, \psi_i')$ in the action, and following algebraic manipulations, we obtain the following:

$$S = \int d^4x\left[\bar{\chi}\left(\Gamma^i\chi_i' - \Gamma^0\dot{\chi}\right) + \frac{1}{4}\left(\Gamma^i\omega_i^{ab} - \Gamma^0\omega_0^{ab}\right)\gamma_{ab}\chi \right. \\ \left. - ma^2\bar{\chi}\chi + \frac{1}{a}\bar{\chi}\left(a_i'\Gamma^i + \dot{a}\Gamma^0\right)\chi\right]. \tag{8}$$

When comparing Equations 6 and 8, we see that the action has developed an extra piece in the form $\frac{1}{a}\bar{\chi}\left(a_i'\Gamma^i + \dot{a}\Gamma^0\right)\chi$. Note that, since $a = a(\eta)$ is not a function of the spatial component of the spacetime, $a_i' = 0$. The extra piece is then just $\frac{\dot{a}}{a}\bar{\chi}\Gamma^0\chi$, and expanding $\Gamma^0 = e_{0a}\gamma^a$ yields $\frac{\dot{a}}{a}\bar{\chi}e_{0a}\gamma^a\chi$. This extra term modifies the equations of motion of the auxiliary spinors. Varying the action functional with respect to the fields $(\chi, \bar{\chi}, e_a^\mu)$ gives us the following set of equations of motion (EOM):

$$\frac{\delta S}{\delta\chi} = \partial_\mu\left(\bar{\chi}\Gamma^\mu\right) + \bar{\chi}\left(\frac{\dot{a}}{a}\Gamma^0 - ma^2\right) = 0, \tag{9}$$

$$\frac{\delta S}{\delta\bar{\chi}} = \Gamma^\mu\partial_\mu\chi + \left(\frac{\dot{a}}{a}\Gamma^0 - ma^2\right)\chi = 0, \tag{10}$$

$$\frac{\delta S}{\delta e_a^\mu} = \bar{\chi}\gamma^a\left(\frac{1}{a}\partial_\mu a + \partial_\mu\chi\right) = 0. \tag{11}$$

When integrating by parts, the variations of the form $\delta(\partial_\mu\chi) = \partial_\mu(\delta\chi)$[12] allow us to disregard the boundary terms as the FLRW universe is asymptotically flat, and the fields $(\chi, \bar{\chi})$ vanish at infinity. In the equations of motion for $(\chi, \bar{\chi})$, the extra terms proportional to $\frac{\dot{a}}{a}\Gamma^0$ cause the Fourier expansions of the EOM solutions to develop time-dependent frequencies $\omega(t)$. However, these time-dependent frequencies pose a challenge as they prevent the definition of a vacuum. Similarly to scalar fields, the expansion of χ into Fourier modes includes creation and annihilation operators $(a_k^\dagger, a_k)$, which are dependent on the frequency of the spinor. This poses a problem because the time-dependent annihilation operators, which annihilate the vacuum at some point in time, do not necessarily annihilate the vacuum at another time. Without a well-defined notion of a vacuum, describing fields or particles in the expanding spacetime becomes impossible. Furthermore, defining an action for a quantum field theory at a given energy scale requires an *effective Wilsonian action*. This is an action whereby irrelevant operators have been removed after integrating out high energy modes. However, if the fields develop time-dependent frequencies, one cannot define an effective Wilsonian action by integrating out high energy spinor modes to keep observables finite. This is because low energy modes can evolve in time to become high energy modes. This is why we cannot define vacuum states over an expanding curved geometry such as de Sitter space.

More problems occur when considering a UV-complete theory that has supersymmetry. It has been shown that over an expanding spacetime, bosonic fields develop time-dependent masses[10]. However, in our case of fermionic fields, their mass remains constant while their frequencies become time-dependent. However, in the supersymmetric framework, the zero-point energy no longer cancels out. This means that we would have to perturb the system about a diverging vacuum energy, meaning we have no reference of finite observable energy. Furthermore, this explicitly breaks supersymmetry, so the theory does not agree with our compactifications coming from type IIB string theory. To prevent these problems, we make use of excited coherent states over supersymmetric Minkowski space, which only break supersymmetry spontaneously, meaning that the vacuum remains supersymmetric.

Excited Coherent States

Here, we alleviate problems associated with fields defined over expanding spacetimes in a UV-complete theory by considering coherent states over supersymmetric Minkowski space. In this framework, we will show that de Sitter space can be a result of compactifications of type IIB string theory and that one calculates a positive cosmological constant for an expanding universe. These coherent states, known as *Glauber-Sudarshan* (GS) states $|\sigma\rangle$, expand and share the isometries of de Sitter space. As these states share the same isometries as the de Sitter space metric, they are of particular interest to replace the problematic interacting de Sitter vacuum states (or *vacua*) $|\Omega\rangle$. The idea is to use these states to compute quantum corrections to the metric *operator* $\hat{g}_{\mu\nu}$, where part of its representation contains an expression for the cosmological constant, which will turn out positive.

We must first consider why coherent states are used in quantum field theory (QFT) and elaborate on the nature of Glauber-Sudarshan states. Additionally, we should understand the nature of the supersymmetric Minkowski space that we are working over. Then, we will move onto considering the metric quantum corrections.

Glauber-Sudarshan (GS) States

For a *free QFT* with a single bosonic Degree of Freedom (DOF) ($\alpha_1 \equiv \alpha$ for example), the coherent state $|\alpha\rangle$ is a shift of the free vacuum $|\alpha\rangle = \mathbb{D}_0(\alpha)\,|0\rangle = \exp\left(\alpha a_k^\dagger - \alpha^* a_k\right)|0\rangle$, where $\mathbb{D}_0(\alpha)$ is the unitary displacement operator, α is a complex number, and $(a_k^\dagger, a_k)$ are the usual creation and annihilation operators. These states are useful because they preserve the degree to which a quantum system exhibits wave-like behavior, such as interference and diffraction. Furthermore, they are the excited quantum states that most closely resemble classic states, with minimal uncertainty in position and momentum. This is useful to us as our theory must reproduce classical physics in certain limits. In the case for an *interacting QFT*, however, with multiple bosonic DOFs $\{\alpha_i\} \equiv \sigma$, we instead have $|\sigma\rangle = \mathbb{D}(\sigma, t)\,|\Omega\rangle$ — the Glauber-Sudarshan states. Being that the “vacuum” state we utilize is now *excited*, there is an ambiguity of the displacement operator and it becomes non-unitary. For the case of interacting vacua, the structure is not as trivial as the free vacua, so preserving the unitarity of the shift operator is difficult. It is however still possible to represent it in terms of the interacting Hamiltonian over a temporal domain as follows[13]:

$$\mathbb{D}(\sigma, t) = \lim_{T\to\infty(1-i\epsilon)} \mathbb{D}_0(\sigma, t)\exp\left(iM_p\int_{-T}^{t} dt\,\mathbf{H}_{\text{int}}\right). \tag{12}$$

Here, $\mathbb{D}_0(\sigma, t)$ is a time-dependent unitary displacement operator for the free vacuum, $\mathbf{H}_{\text{int}}$ is the interaction Hamiltonian of the theory, M_p is the Planck mass, and the limit is slightly in the imaginary direction $(1 - i\epsilon)$ for the same reason as it is for propagators: to avoid poles in the complex phase that give rise to divergences in observables. Now, for reasons which will become clear later, we must *uplift* type IIB string theory to *M-theory*, an 11D string theory (we can recover type IIB string theory by compactifying along the 11th dimension in M-theory). The multiplet for the fields in M-theory is given by $(g_{ab}, C_{abc}, \psi_a)$, where g_{ab} is the metric field, C_{abc} is 3-form field, and ψ_a is a vector-spinor. One can imagine describing all the DOFs of the respective fields via a collection of numbers (in the form of a vector / matrix / higher-order construction). The DOFs of the metric field g_{ab} are captured by the set $\{\alpha_{ab}\}$ for some numbers α_{ab}, the set $\{\beta_{abc}\}$ for some numbers β_{abc} captures the DOF of the 3-form field C_{abc}, and finally the set $\{\gamma_a\}$ for some numbers γ_a captures the DOF of the vector-spinor field ψ_a. Thus, fields in the M-theory multiplet respectively come with collective DOFs $(\{\alpha_{ab}\}, \{\beta_{abc}\}, \{\gamma_a\})$ (128 from the bosonic sector and 128 from the fermionic sector, totaling 256 DOFs). Denoting all the DOFs as $\sigma \equiv (\{\alpha_{ab}\}, \{\beta_{abc}\}, \{\gamma_a\})$, we write the coherent Glauber-Sudarshan state as[14]:

$$|\sigma\rangle \equiv \mathbb{D}(\sigma, t)\,|\Omega\rangle = \bigotimes_k \left(\sum_{f_k} \Psi^\sigma(f_k)\,|f_k\rangle\right). \tag{13}$$

Here, the sum is over all the fields $f_k = (\{g_{ab}(k)\}, \{C_{abc}(k)\}, \{\psi_a(k)\})$ and the tensor product is over the mode momenta k. Ψ^σ is the wave function of the GS state (corresponding to the DOFs of the field f_k given by σ) which can be seen as a product of normalized Dirac delta functions, and $|f_k\rangle$ are eigenstates of the momentum wavefunction of the GS states in the configuration space. This is a general state that contains all 256 DOFs of the field in M-theory and is coherent. This is the description of the states we will use for the quantum corrections.

Supersymmetric Minkowski Space

Since type IIB theory is a 9+1D theory, we must compactify on a 6D manifold to recover our 3+1D universe (the de Sitter vacuum solution). For explicit calculations, it is useful to pick a *slicing* of de Sitter space (a selection of a foliation). We choose one which makes de Sitter space appear flat within a certain region, given by $ds^2 = 1/(\Lambda|t|^2)\eta_{\mu\nu}dx^\mu \otimes dx^\nu$. Here, t is the conformal time coordinate (instead of writing η), Λ is the cosmological constant, $\eta_{\mu\nu}$ is the usual Minkowski metric, and dx^μ is the basis of the contangent space. A flat slicing gives us a temporal domain $-1/\sqrt{\Lambda} \le t < 0$, over which the metric is well defined, where $t = 0$ represents late times. This temporal domain comes precisely from the trans-Planckian censorship conjecture (TCC) time scale[15]. Although we are working with supersymmetric Minkowski space (a superspace), it suffices for our analyses to look at the non-supersymmetric part of the 10D space. In this case, the full space is given by:

$$\mathcal{M}_{10} = \mathbb{R}^{3,1} \times \mathcal{M}_4 \times \mathcal{M}_2, \tag{14}$$

where $\mathbb{R}^{3,1}$ is 3+1D Minkowski space, and $\mathcal{M}_4 \times \mathcal{M}_2 \equiv \mathcal{M}_6$ is some non-Kähler 6D internal space (written as a product space to account for multiple scaling factors for different pieces of the internal space). Moreover, to align with more general geometries that include warping, we define the metric of the 10D space as the warped geometry[13]:

$$ds^2 = \frac{1}{\Lambda H^2(y)|t|^2}\eta_{\mu\nu}dx^\mu \otimes dx^\nu + H^2(y)\left[F_1(t)g_{\alpha\beta}dy^\alpha \otimes dy^\beta + F_2(t)g_{mn}dy^m \otimes dy^n\right]. \quad (15)$$

Here, $H(y)$ is the *warp factor* that depends on the internal space coordinates $\{y^m, y^\alpha\}$, and which makes the external space explicitly dependent on the internal space's behaviour. $\{x^\mu\}$ are the coordinates of the external space $\mathbb{R}^{3,1}$. This makes us no longer consider a local quantum field theory as it includes non-local interactions. Furthermore, $(F_1(t), F_2(t))$ are scaling factors of the different internal subspaces $(\mathcal{M}_4, \mathcal{M}_2)$, respectively, equipped with metrics $(g_{\alpha\beta}, g_{mn})$. When computing quantum corrections to the metric operator, we compute path integrals over the flat metric to reproduce the above metric. Path integrals require a full description of the system's action, however, there are currently *no* well-defined actions for type IIB string theory[16]. To work with a string theory that has a well-defined action, we consider the uplift of type IIB string theory to M-theory, an 11D string theory. As mentioned in the previous section, the multiplet for M-theory is $(g_{\mu\nu}, C_{\mu\nu\rho}, \psi_\mu)$, and the 10D space now becomes an 11D space given by the following metric:

$$\mathcal{M}_{11} = \mathbb{R}^{3,1} \times \mathcal{M}_4 \times \mathcal{M}_2 \times \frac{\mathbb{T}^2}{\mathbb{Z}_2}. \quad (16)$$

Notice that the extra 11th dimension appears as the quotient space between the torus $\mathbb{T}^2$ and the $\mathbb{Z}_2$ (the group of integers mod 2). We define coordinates of the space as $(x^\mu, x^\nu) = (x^0, \dots, x^3)$, $(y^m, y^n) = (x^4, \dots, x^9)$, and $(\omega^a, \omega^b) = (x^{10})$, for the spaces $(\mathbb{R}^{3,1}, \mathcal{M}_6, \frac{\mathbb{T}^2}{\mathbb{Z}_2})$ respectively. This modifies the overall metric of the 11D space, the explicit form for which is shown in Alexander *et al.*[16] For clarity, we also consider indices $(M, N) = (0, \dots, 10)$, which takes into account the information (coordinates) of the entire space and will work with the multiplet $(g_{MN}, C_{MNP}, \psi_M)$ defined over all of $\mathcal{M}_{11}$. For explicit computations with this group of fields, the overall multiplet can be decomposed over the different sub-pieces of the spacetime as direct sums of singlets (this is known as a *dimensional reduction*).

Quantum Corrections

Now that we have an expression for the space we are working with and have defined the GS states, we move onto computing the quantum corrections in the form of contributions affecting the metric *operator* $\hat{g}_{\mu\nu}$. To note, the explicit reference to *operators* addresses the subtlety between operators and fields when making use of path integrals. The correction takes form of the expectation value $\langle\sigma|\, \hat{g}_{\mu\nu}\, |\sigma\rangle$ normalized by $\langle\sigma|\sigma\rangle$ (defined as $\langle\hat{g}_{\mu\nu}\rangle_\sigma$). The expression for the correction is a quotient of path integrals (functional integrals over all possible evolutions of the fields) over the M-theory multiplet in 11D (ref. 16) (denoted as $\langle\hat{g}_{\mu\nu}\rangle_\sigma$):

$$\frac{\int \mathcal{D}[g_{MN}]\mathcal{D}[C_{MNP}]\mathcal{D}[\psi_M]\mathcal{D}[\bar{\psi}_N]\; e^{iS}\; \mathbb{D}^\dagger(\sigma,t)g_{\mu\nu}(x)\mathbb{D}(\sigma,t)}{\int \mathcal{D}[g_{MN}]\mathcal{D}[C_{MNP}]\mathcal{D}[\psi_M]\mathcal{D}[\bar{\psi}_N]\; e^{iS}\; \mathbb{D}^\dagger(\sigma,t)\mathbb{D}(\sigma,t)}. \quad (17)$$

Here, $\mathcal{D}[A_{M\dots N}] \sim \prod_{M,\dots,N} dA_{M\dots N}$ are the path integral field measures (where $A_{M\dots N}$ represents the different fields of the multiplet that are integrated over), S is the total action of the system, $\mathbb{D}^\dagger(\sigma,t)$ is the non-unitary shift operator, and $g_{\mu\nu}(x)$ is the metric of Minkowski space $\mathbb{R}^{3,1}$. This path integral is much too complicated at this level of generality with Grassmanian integrals (coming from integrals over the vector-spinors $\mathcal{D}[\psi_M]\mathcal{D}[\bar{\psi}_N]$), so we picked three representative sample scalars $(\varphi_1, \varphi_2, \varphi_3)$ for each of the DOFs of the fields in the M-theory multiplet. Theses scalars fix the DOFs to $\sigma \equiv (\varphi_1, \varphi_2, \varphi_3)$, and so for the DOFs φ_1 that contribute towards the metric operator correction, we have the replacement $\langle\hat{g}_{\mu\nu}\rangle_\sigma \to \langle\varphi_1\rangle_\sigma$. Although one might assume the expectation value of a scalar to vanish, in this case we are still working with the different fields in the M-theory multiplet, so have simply *reduced* their DOFs to that of scalars. Once fixed, the path integrals in the numerator and denominator of $\langle\varphi_1\rangle_\sigma$ fall under a class of path integrals that can be computed using *nodal diagrams*, due to the shifted-vacuum structure of the GS state. The shifted-vacuum structure is in reference to the GS state being a non-trivial shift of the vacuum state $|0\rangle$ to the GS state $|\sigma\rangle$ via the non-unitary shift operator $\mathbb{D}(\sigma,t)$. This shift structure allows us to make use of nodal diagrams, which is the set of Feynman diagrams that capture the information of higher order point functions[14]. These diagrams emerge from how the different momenta of the fields within the multiplet are summed over. Using the amplitudes of the nodal diagrams $\{\mathcal{A}_s\}$, it is possible to express the quantum correction as the following[14] (denoted as $\langle\varphi_1\rangle_\sigma$):

$$\frac{\text{TLN} + \sum_{n,\dots,s} c_{mnpqrs}\, \mathcal{N}^{(1)}_{nmp}(k;q) \otimes \mathcal{N}^{(2)}_{nmp}(l;r) \otimes \mathcal{N}^{(3)}_{nmp}(f;s)}{\text{TLD} + \sum_{n,\dots,s} c_{mnpqrs}\, \mathcal{N}^{(1)'}_{nmp}(k;q) \otimes \mathcal{N}^{(2)}_{nmp}(l;r) \otimes \mathcal{N}^{(3)}_{nmp}(f;s)}. \quad (18)$$

Here, (TLN, TLD) are the tree-level Feynman diagram contributions to the numerator and denominator path integrals, respectively. The contributions of Feynman diagrams with loops comes from summing over the amplitudes of the nodal diagrams. Here, $\mathcal{N}^{(i)}_{nmp}(a;b)$ represents the Feynman diagram amplitude (nodal diagram amplitude) of interactions between fields $\varphi^b_i(a)$, where i labels the different scalars that represent the DOFs of the multiplet in M-theory $(\varphi_1, \varphi_2, \varphi_3)$, a labels the incoming momenta of the fields, and b is an integer. The total loop contribution comes from summing over the nodal amplitudes for different i, which is weighted by the coupling constants c_{nmpqrs}. Furthermore, the prime in $\mathcal{N}^{(1)'}_{nmp}(k;q)$ refers to interaction diagrams without field sources. These nodal diagrams diverge with structure of the Gevrey kind (meaning they diverge factorially) and require Borel resummation[‡] to restructure the divergence into non-perturbative solitonic corrections. Using the nodal amplitudes and Borel re-summation gives the following correction to the metric operator[16]:

$$\langle\varphi_1\rangle_\sigma = \sum_{\{s\}}\left[\frac{1}{g^{1/l}_{(s)}}\int_0^\infty d\mathcal{B}\exp\left(-\frac{\mathcal{B}}{g^{1/l}_{(s)}}\right)\frac{1}{1-\mathcal{A}_{(s)}\mathcal{B}^l}\right]_{P.V} \times \int_{k_{\text{IR}}}^{\mu} d^{11}k\; \frac{\bar{\alpha}_{\mu\nu}(k)}{a(k)}\; \mathbf{Re}\left(\psi_k e^{-i(k_0-\bar{\kappa}_{\text{IR}})t}\right), \quad (19)$$

where $\{s\}$ is the set of interactions, $g_{(s)}$ is the set of coupling constants, $\mathcal{B}$ parametrizes an axis in the Borel plane, $\mathcal{A}_{(s)}$ is the amplitude of all possible nodal diagrams, $P.V$ is the principal value of the integral over $\mathcal{B}$, $(k_{\text{IR}}, \bar{\kappa}_{\text{IR}})$ are IR scales, $\bar{\alpha}_{\mu\nu} = \alpha_{\mu\nu}/V$ is a 2-form normalized by the volume V of the space $\mathcal{M}_{11}$, $a(k) = k^2/V$, l is the total amount of fields minus one, k is the momentum (which has an associated k_0 component), and ψ_k is the spatial wavefunction of the GS state over the space $\mathcal{M}_{11}$ (projecting the GS state $|\sigma\rangle$ into the coordinate space of $\mathcal{M}_{11}$). As before, $(\mu,\nu) = (0,\dots,3)$

‡A resummation is a prescription to make a specific class of divergent series convergent via regularization and rescaling.

labels the spacetime coordinates of the external space $\mathbb{R}^{3,1}$. It turns out that the first piece of the form $\sum[\dots]$ exactly corresponds to the inverse of the cosmological constant (at a given IR scale κ)[14], which is defined over the same time domain as the GS state ($-\frac{1}{\sqrt{\Lambda}} < t \leq 0$). We then have:

$$\frac{1}{\Lambda^\kappa} \equiv \sum_{\{s\}} \left[\frac{1}{g_{(s)}^{1/l}} \int_0^\infty d\mathcal{B} \exp\left(-\frac{\mathcal{B}}{g_{(s)}^{1/l}}\right) \frac{1}{1-\mathcal{A}_{(s)}\mathcal{B}^l} \right]_{P.V}, \quad -\frac{1}{\sqrt{\Lambda}} < t \leq 0. \tag{20}$$

This is known as the *integral form* of the cosmological constant Λ^κ, and its integral in this case is **positive definite** over the flat slicing of the temporal domain[16]. This means that the constant affords a positive sign — the desired sign of the cosmological constant. To recover the result for type IIB string theory from M-theory, one need only compactify along a compact direction, which leaves the above form of the cosmological constant unchanged. This shows that using the GS states (which have the isometries of Minkowski space and preserve supersymmetry) yields the correct sign for the cosmological constant. Here, the cosmological constant functional expression is issued from the quantum corrections of the metric operator, and its correct positive sign indicates that we indeed have compactifications of the de Sitter vacuum solution from type IIB string theory. Hence, with the combination of GS states and Minkowski space, we can consider de Sitter space as a candidate of our universe in a UV-complete theory (as coming from a type IIB string theory compactifications).

Conclusion

The swampland scenario prevents expanding spaces with positive curvature from producing a cosmological constant with the correct sign (i.e., positive), whether in UV-complete or incomplete theories. Furthermore, in the presence of an expanding spacetime, fields (and hence the particles they describe) develop time-dependent frequencies, meaning particles are not well defined.

To counteract this, we considered a candidate for our universe, de Sitter space, in a UV-complete theory whereby we recovered the de Sitter vacuum solution through compactifications of type IIB string theory. To avoid the ambiguity of de Sitter vacuum states over an expanding spacetime, we reviewed work done on a class of general coherent states (known as Glauber-Sudarshan states, which have the desired de Sitter isometries) over supersymmetric Minkowski space. We showed that we can define these states over a given temporal domain via the non-unitary shift operator which excites the interacting vacua to give the GS states. These states allowed us to compute the metric operator quantum corrections, which contains an expression for the cosmological constant. We showed that the constant is positive definite, meaning it has the correct sign for an expanding universe. This shows that we can have consistent compactifications to de Sitter space in type IIB string theory, and thus the de Sitter vacuum solution can be used as a candidate for our universe.

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